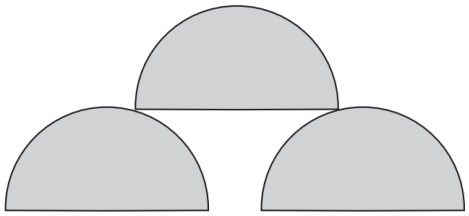
Surface Areas and Volumes

Case Study Based Questions

Case Study 1

Priyansh had three hemispherical shaped boxes. He thought of putting down his stationary in one of these boxes, so that it does not get lost. The radius of each hemispherical box is 7 cm.



On the basis of the above information, solve the following questions:

Q1. The capacity of a box is:

a. 730 cm^3 b. 1500 cm^3

c. 718.67 cm^3 d. 1900 cm^3

Q2. If any two box is melted and recasted into a single sphere, then radius of sphere will be:

a. 3.5 cm b. 7 cm

c. 14 cm d. 10.5 cm

Q3. The base area of a box is:

a. 150 cm^2 b. 180 cm^2

c. 154 cm^2 d. 170 cm^2

Q4. The total surface area of a box is:

a. 340 cm^2 b. 308 cm^2

c. 380 cm^2 d. 830 cm^2

Q5. If the cost of painting a box per cm^2 is ₹ 2, then the total cost of painting a box is:

a. 740 b. 750

c. 730 d. 616

Solutions

1. (c) Given, radius, $r = 7$ cm
$\therefore$ The capacity of a box = Volume of hemispherical shaped box

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (7)^3 = \frac{2156}{3}$$

$$= 718.67 \text{ cm}^3$$

So, option (c) is correct.

2. (b) Given, 2× Volume of hemisphere = Volume of sphere

$$\Rightarrow \quad 2 \times \frac{2}{3} \times \pi \times (7)^3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad 7^3 = R^3$$

$$\Rightarrow \quad R = 7\text{cm}$$

Hence, radius of required sphere is 7cm.
So, option (b) is correct.

3. (c) The base area of a box $= \pi r^2$

$$= \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$$

So, option (c) is correct.

4. (b) The total surface area of a box $= 2\pi r^2$

$$= 2 \times 154$$

$$= 308 \text{ cm}^2$$

So, option (b) is correct.

5. (d) Given, 1 cm^2 area is painted in ₹ 2.
Therefore, 308 cm^2 area is painted in ₹ 2×308

$$= ₹\ 616.$$

So, option (d) is correct.

Case Study 2

During vacation, two friends decided to visit Shimla. Due to peak days in Shimla, they did not get any room to stay it. So, they thought to buy a tent and set up in a park. They made a tent in the shape of cone, whose diameter is 14 cm and height is 22 cm.

On the basis of the above information, solve the following questions:

Q1. How much volume of air is stored in a conical tent?

a. 1140 cm^3 b. 1132 cm^3

c. 1129.33 cm^3 d. 1134 cm^3

Q2. The slant height of a cone is:

a. $\sqrt{521}$ cm b. $\sqrt{533}$ cm

c. $2\sqrt{533}$ cm d. $\sqrt{537}$ cm

Q3. The base area covered by a conical tent is:

a. 150 cm^2 b. 153 cm^2

c. 154 cm^2 d. 159 cm^2

Q4. The curved surface area of a conical tent is:

(Use $\sqrt{533} = 23.09$)

a. 508 cm^2 b. 507.98 cm^2

c. 509 cm^2 d. 512 cm^2

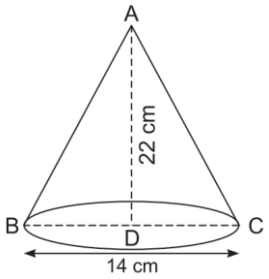
Q5. If the cost of painting a tent is 50 paise per cm^2, then the cost of painting the tent is (approximate):

a. 260 b. 255

c. 254 d. 270

Solutions

1. (c) Given, diameter of cone, d = 14 cm and height of cone, h = 22 cm



Therefore, radius of a cone, $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

$\therefore$ Volume of air stored in a conical tent

= Volume of cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 22 \text{ cm}^3$$

$= 1129.33 \text{ cm}^3$

So, option (c) is correct.

2. (b)

$\therefore$ The slant height of a cone $= \sqrt{(7)^2 + (22)^2}$

$$= \sqrt{49 + 484} = \sqrt{533} \text{ cm}$$

So, option (b) is correct.

3. (c) The base area covered by a conical tent is

A = Area of circle $= \pi r^2$

$$= \frac{22}{7} \times (7)^2 = 22 \times 7 = 154 \text{ cm}^2$$

So, option (c) is correct.

4. (b) The curved surface area of a conical tent $= \pi rl$

$$= \frac{22}{7} \times 7 \times \sqrt{533} \qquad [\because l = \sqrt{533} \text{ cm}]$$

$= 22 \times 23.09 = 507.98 \text{ cm}^2$

So, option (b) is correct.

5. (c) Given, 1 cm^2 is painted for 50 paise or ₹ 0.5.

$\therefore$ 507.98 cm^2 is painted for ₹ 507.98×0.5

$= ₹\ 253.99 \approx ₹\ 254$

So, option (c) is correct.

Case Study 3

For decoration purpose, Sneha bought 100 orbeez balls and put it in a cylindrical shaped box. After filling it with water, the orbeez ball swell up and completely filled the cylindrical shaped box. Behind the orbeez ball packet, the change in volume of each orbeez ball was mentioned and which was 32% increase. Suppose the volume of all orbeez ball is 9900 cm^3.

On the basis of the above information, solve the following questions:

Q1. The volume of each orbeez ball is:

a. 97 cm^3 b. 99 cm^3

c. 96 cm^3 d. 94 cm^3

Q2. Volume of orbeez ball before swelling is:

a. 70 cm^3 b. 78 cm^3

c. 75 cm^3 d. 85 cm^3

Q3. How many orbeez balls before swelling was needed to completely fill the cylindrical shaped box?

a. 132 b. 138

c. 134 d. 140

Q4. What is the cubic radius of an orbeez ball before swelling up?

a. 17.30 cm^3 b. 16.50 cm^3

c. 17.70 cm^3 d. 17.89 cm^3

Q5. If the change in volume of orbeez ball is increased to 48%, then the volume of orbeez ball after swell up will be:

a. 114 cm^3 b. 111 cm^3

c. 112 cm^3 d. 116 cm^3

Solutions

1. (b) Volume of 1 orbeez ball × 100 = Volume of all orbeez ball

$\therefore$ Volume of one orbeez ball $= \frac{9900}{100} = 99\ cm^3$

So, option (b) is correct.

2. (c) Let original volume of orbeez ball be x.

Change in volume of orbeez ball

$$= \frac{99 - x}{x} \times 100$$

Given, $32 = \frac{(99 - x) \times 100}{x}$

$\Rightarrow$ $32x = 9900 - 100x$

$\Rightarrow$ $132\ x = 9900$

$\Rightarrow$ $x = \frac{9900}{132}$

$\therefore$ $x = 75\ cm^3$

So, option (c) is correct.

3. (a) n x Volume of 1 orbeez ball before swell up

= Volume of container

$\Rightarrow$ n x 75 cm^3 = 9900 cm^3

$\Rightarrow$ $n = \frac{9900}{75}$

$\therefore$ n = 132 balls

So, option (a) is correct.

4. (d) Volume of an orbeez ball before swell up = 75 cm^3

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 75$$

$$\Rightarrow \quad \frac{4}{3}\times\frac{22}{7}\times r^3 = 75$$

$$\Rightarrow \quad r^3 = \frac{75\times 3\times 7}{4\times 22}$$

$$\therefore \quad r^3 = 17.89 \text{ cm}^3$$

So, option (d) is correct.

5. (b) Let the volume of orbeez ball after swell up be x.

Then, $$48 = \frac{x-75}{75}\times 100$$

$$\Rightarrow \quad 48 = \frac{x-75}{3}\times 4$$

$$\Rightarrow \quad 48 \times 3 = 4x - 300$$

$$\Rightarrow \quad 4x = 144 + 300 \Rightarrow 4x = 444 \text{ cm}^3$$

$$\therefore \quad x = 111 \text{ cm}^3$$

So, option (b) is correct.

Case Study 4

Students of class IX were taken to an educational trip to Delhi. They were first shown Humayun's Tomb. It was the tomb of Mughal Emperor Humayun and is a great example of Mughal Architecture and it is an UNESCO approved World Heritage site. Since, it was a very old monument restoration and paint was going on. Maths teacher decided to ask some questions. He told them in the figure a dome can be seen, when radius of the dome is 15 m and, in the monument, there can be seen 2 pillars which resembles a cylinder whose radius and height is 28 m and 47 m.

On the basis of the above information, solve the following questions:

Q1. Find the curved surface area of the dome.

Q2. Find the volume of the dome.

Solutions

1. The dome resembles a hemispherical sphere

Given, radius = 15 m

Curved surface area of the hemispherical sphere

$= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 15 \times 15 = \frac{9900}{7} \text{ m}^2$$

$$= 1414.285 \text{ m}^2$$

2. Volume of the dome = Volume of hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 15 \times 15 \times 15 = \frac{49500}{7}$$

$$= 7071.43 \text{ m}^3$$